LOGARITMOS

A. Introducción Teoría

B. Ejercicios resueltos

A. INTRODUCCIÓN TEÓRICA

A.1 Definición de logaritmo:

Sea x un número. El logaritmo de ese número es el exponente al que hay que elevar cierta base b para obtener x:

$$x = b^y \Leftrightarrow y = \log_b x$$

Ejemplo:

- El logaritmo de 16 en base 2 es el exponente al que hay que elevar la base 2 para obtener 16, es decir, cuatro:

$$\log_2 16 = 4 \text{, ya que } 16 = 2^y \Leftrightarrow y = \log_2 16 = 4$$

A.2 Logaritmos naturales:

Los logaritmos que tienen como base al número e, son llamados “logaritmos naturales”. Se simbolizan con la abreviatura **ln**.

$$\ln x = \log_e x$$

A.3 Cambio de base en los logaritmos:

Si queremos expresar $\log_a x$ mediante $\log_b x$ sólo tenemos que tener en cuenta que:

$$\log_b M = \frac{\log_a M}{\log_a b}$$

A.4 Propiedades:

I.	$\log_a MN = \log_a M + \log_a N$	IV.	$\log_a 1 = 0$
II.	$\log_a M^p = p \cdot \log_a M$	V.	$\log_a a = 1$
III.	$\log_a \frac{M}{N} = \log_a M - \log_a N$	VI.	$a^{\log_a b} = b$

B. EJERCICOS RESUELTOS

B.1. Dado un logaritmo, halla su valor:

1. $\log_2 64 = \log_2 2^6 = 6 \cdot \log_2 2 = 6 \cdot 1 = 6$

2. $\log_2 \sqrt{2} = \log_2 2^{\frac{1}{2}} = \frac{1}{2} \cdot \log_2 2 = \frac{1}{2} \cdot 1 = \frac{1}{2}$

3. $\log_{\frac{1}{2}} \sqrt{2} = \log_{\frac{1}{2}} 2^{\frac{1}{2}} = \frac{1}{2} \cdot \log_{\frac{1}{2}} 2 = \frac{1}{2} \cdot \log_{\frac{1}{2}} \left(\frac{1}{2}\right)^{-1} = \frac{1}{2} \cdot (-1) \log_{\frac{1}{2}} \left(\frac{1}{2}\right) = -\frac{1}{2}$

4. $\log_{\frac{1}{3}} \sqrt[5]{81} = \log_{\frac{1}{3}} \sqrt[5]{3^4} = \log_{\frac{1}{3}} \left(3^4\right)^{\frac{1}{5}} = \log_{\frac{1}{3}} 3^{\frac{4}{5}} = \frac{4}{5} \cdot \log_{\frac{1}{3}} 3 = \frac{4}{5} \cdot \log_{\frac{1}{3}} \left(\frac{1}{3}\right)^{-1} =$

$= \frac{4}{5}(-1) \cdot \log_{\frac{1}{3}} \left(\frac{1}{3}\right) = -\frac{4}{5} \cdot 1 = -\frac{4}{5}$

5. $\log_{10}\left(5\log_{10}100\right)^2 = 2\log_{10}\left(5\log_{10}100\right) = 2\log_{10}\left(5\log_{10}10^2\right) =$
$= 2\log_{10}\left(5\cdot 2\log_{10}10\right) = 2\log_{10}10 = 2$

6. $\log_{\sqrt{2}} 32 = \log_{2^{\frac{1}{2}}} 2^5 = \log_{2^{\frac{1}{2}}}\left(2^{\frac{1}{2}}\right)^{10} = 10\cdot\log_{2^{\frac{1}{2}}}\left(2^{\frac{1}{2}}\right) = 10$

7. $\log_{9\sqrt{3}} 3\cdot\sqrt[5]{27} = \log_{3\cdot 3^{\frac{1}{2}}} 3\cdot\left(3^3\right)^{\frac{1}{5}} = \log_{3^{\frac{1}{2}+1}} 3^{\frac{3}{5}+1} = \log_{3^{\frac{3}{2}}} 3^{\frac{8}{5}} = \log_{3^{\frac{3}{2}}}\left(3^{\frac{3}{2}}\right)^{\frac{8}{5}\cdot\frac{2}{3}} =$
$= \frac{16}{15}\log_{3^{\frac{3}{2}}}\left(3^{\frac{3}{2}}\right) = \frac{16}{15}$

B.2. Dada una expresión logarítmica, hallar su valor.

8. $\log_2\sqrt[5]{2} + \log_2 8 + \log_2\frac{1}{4} = \log_2 2^{\frac{1}{5}} + \log_2 2^3 + \log_2 2^{-2} =$
$= \frac{1}{5}\log_2 2 + 3\log_2 2 - 2\log_2 2 = \frac{1}{5} + 3 - 2 = \frac{6}{5}$

9. $\ln 1 + \ln e + \ln e^3 + \ln\sqrt[3]{e} + \ln\frac{1}{e} = = 0 + 1 + 3\ln e + \ln e^{\frac{1}{3}} + \ln e^{-1} =$

10. $\log 810 + \log 0{,}03 + \log\sqrt[5]{\frac{1}{9}}$, si $\log 3 \approx 0{,}477$

$\log 810 + \log 0{,}03 + \log\sqrt[5]{\frac{1}{9}} = \log\left(3^4\cdot 10\right) + \log\frac{3}{100} + \log\left(3^{-2}\right)^{\frac{1}{5}} =$
$= 4\log 3 + \log 10 + \log 3 - \log 10^2 - \frac{2}{5}\log 3 = 4\log 3 + 1 + \log 3 - 2 - \frac{2}{5}\log 3 =$
$= \frac{23}{5}\log 3 - 1 = 2{,}1942 - 1 = 1{,}1942$

11. $\log\sqrt[5]{0{,}04} + \log\sqrt[3]{\frac{0{,}25}{8}} + \log\sqrt{\frac{1{,}6}{5}}$, si $\log 2 \approx 0{,}301$

$\log\sqrt[5]{0{,}04} + \log\sqrt[3]{\frac{0{,}25}{8}} + \log\sqrt{\frac{1{,}6}{5}} =$
$= \log\left(\frac{2^2}{100}\right)^{\frac{1}{5}} + \log\left(\frac{\frac{5^2}{100}}{2^3}\right)^{\frac{1}{3}} + \log\left(\frac{\frac{2^4}{10}}{5}\right)^{\frac{1}{2}} =$

$$=\frac{1}{5}\log\left(\frac{2^2}{100}\right)+\frac{1}{3}\log\left(\frac{\frac{5^2}{100}}{2^3}\right)+\frac{1}{2}\log\left(\frac{\frac{2^4}{10}}{5}\right)=$$

$$=\frac{1}{5}\left(\log 2^2-\log 10^2\right)+\frac{1}{3}\left[\log\left(\frac{5^2}{100}\right)-\log 2^3\right]+\frac{1}{2}\left[\log\left(\frac{2^4}{10}\right)-\log 5\right]=$$

$$=\frac{1}{5}(2\log 2-2\log 10)+\frac{1}{3}\left[2\log\frac{10}{2}-2\log 10-3\log 2\right]+\frac{1}{2}\left[4\log 2-\log 10-\log\frac{10}{2}\right]=$$

$$=\frac{1}{5}(2\log 2-2)+\frac{1}{3}\left[2(1-\log 2)-2-3\log 2\right]+\frac{1}{2}\left[4\log 2-1-(1-\log 2)\right]=$$

$$=\frac{2}{5}\log 2-\frac{2}{5}+\frac{2}{3}-\frac{2}{3}\log 2-\frac{2}{3}-\log 2+2\log 2-\frac{1}{2}-\frac{1}{2}-\frac{1}{2}\log 2=-\frac{7}{5}+\frac{7}{30}\log 2=$$

$$=-\frac{7}{5}+\frac{7}{30}\log 2=-1{,}33$$

12. $$\log_a a\sqrt[5]{a}+\log_{\frac{1}{a}}\frac{\sqrt[3]{a}}{\sqrt{a}}=\log_a a\cdot a^{\frac{1}{5}}+\log_{\frac{1}{a}}\sqrt[6]{\frac{a^2}{a^3}}=\log_a a^{\frac{6}{5}}+\log_{\frac{1}{a}}\left(\frac{1}{a}\right)^{\frac{1}{6}}=$$

$$=\frac{6}{5}+\frac{1}{6}=\frac{41}{30}$$

13. $$\log_{a-b}\sqrt[3]{\frac{1}{a-b}}+\log_{\frac{a}{b}}\frac{b}{a}+\log_{a+b}\sqrt{a+b}=$$

$$=\log_{a-b}(a-b)^{\frac{1}{3}}+\log_{\frac{a}{b}}\left(\frac{a}{b}\right)^{-1}+\log_{a+b}(a+b)^{\frac{1}{2}}=$$

$$=\frac{1}{3}\log_{a-b}(a-b)-\log_{\frac{a}{b}}\left(\frac{a}{b}\right)+\frac{1}{2}\log_{a+b}(a+b)=\frac{1}{3}-1+\frac{1}{2}=-\frac{1}{6}$$

14. $$\log_a\left(\sqrt[3]{a}\cdot a^3\right)-\log_b\left(\sqrt[5]{b^2}:b^2\right)+\log_{a\cdot b}(ab)^{-3}=$$

$$=\log_a\left(a^{\frac{1}{3}}\cdot a^3\right)-\log_b\left(\sqrt[5]{b^{-8}}\right)-3=\log_a\left(a^{\frac{10}{3}}\right)-\log_b\left(b^{\frac{-8}{5}}\right)-3=\frac{10}{3}+\frac{8}{5}-3=\frac{29}{15}$$

15. $$\frac{\log_{a-b}\sqrt{\frac{1}{a-b}}+\log_{\frac{a}{b}}\frac{b}{a}}{\log_{a+b}\sqrt{a+b}}=\frac{\log_{a-b}(a-b)^{-\frac{1}{2}}+\log_{\frac{a}{b}}\left(\frac{a}{b}\right)^{-1}}{\log_{a+b}(a+b)^{\frac{1}{2}}}=$$

$$=\frac{-\frac{1}{2}\log_{a-b}(a-b)-\log_{\frac{a}{b}}\left(\frac{a}{b}\right)}{\frac{1}{2}\log_{a+b}(a+b)}=\frac{-\frac{1}{2}-1}{\frac{1}{2}}=-3$$

16. $$\frac{\log_2\sqrt[5]{8}+\log_2 16+\log_2\frac{1}{8}}{2\log_2 4-3\log_2 2}=\frac{\log_2\left(2^3\right)^{\frac{1}{5}}+\log_2 2^4+\log_2 2^{-3}}{\log_2 4^2-\log_2 2^3}=$$

$$=\frac{\log_2 2^{\frac{3}{5}}+4\log_2 2-3\log_2 2}{4\log_2 2-3\log_2 2}=\frac{\frac{3}{5}+4-3}{4-3}=\frac{8}{5}$$

17. $$\frac{\log_2 8+\log_2\frac{2}{25}}{\log_2 40-\log_2 10}-\frac{\log_2\frac{1}{5}-\log_2\frac{25}{8}}{\log_2 2+\log_2 4}=$$

$$=\frac{\log_2 2^3+\left(\log_2 2-\log_2 5^2\right)}{\left(\log_2 5+\log_2 2^3\right)-\left(\log_2 5+\log_2 2\right)}-\frac{\log_2 5^{-1}-\left(\log_2 5^2-\log_2 2^3\right)}{\log_2 2+\log_2 2^2}=$$

$$=\frac{3+\left(1-2\log_2 5\right)}{\left(\log_2 5+3\right)-\left(\log_2 5+1\right)}-\frac{-\log_2 5-\left(2\log_2 5-3\right)}{1+2}=$$

$$=\frac{4-2\log_2 5}{2}-\frac{-3\log_2 5+3}{3}=2-\log_2 5-1+\log_2 5=1$$

18. $\log_b\left(\dfrac{7,2^3\cdot 0,006^{-2}}{25\cdot 3,2^4}\right)$. Datos: $\begin{cases}\log_b 2=4\\ \log_b 3=2\\ \log_b 5=-3\end{cases}$

$$\log_b\left(\frac{7,2^3\cdot 0,006^{-2}}{25\cdot 3,2^4}\right)=\log_b\left(7,2^3\cdot 0,006^{-2}\right)-\log_b\left(25\cdot 3,2^4\right)=$$

$$=\log_b\left(\frac{2^2\cdot 3^2}{5}\right)^3+\log_b\left(\frac{3}{2^2\cdot 5^3}\right)^{-2}-\left[\log_b\left(5^2\right)+\log_b\left(\frac{2^4}{5}\right)^4\right]=$$

$$=3\left[2\log_b 2+2\log_b 3-\log_b 5\right]-2\left[\log_b 3-2\log_b 2-3\log_b 5\right]-$$

$$-\left[2\log_b 5+4\left(4\log_b 2-\log_b 5\right)\right]=$$

$$=3\left[2\cdot 4+2\cdot 2-(-3)\right]-2\left[2-2\cdot 4-3(-3)\right]-\left\{2(-3)+4\left[4\cdot 4-(-3)\right]\right\}=$$

$$=3[8+4+3]-2[2-8+9]-\left\{-6+4[16+3]\right\}=-31$$

B.3 Hallar el término desconocido.

19. $\log_a 125=3\Rightarrow a^3=125\Rightarrow a^3=5^3\Rightarrow a=5$

20. $\log_a 243=5\Rightarrow a^5=243\Rightarrow a^5=3^5\Rightarrow a=3$

21. $\log_{625} 25=x\Rightarrow 625^x=25\Rightarrow 5^{4x}=5^2\Rightarrow 4x=2\Rightarrow x=\frac{1}{2}$

22. $\log_{32} 0,25=x\Rightarrow 32^x=0,25\Rightarrow 2^{5x}=\frac{1}{4}\Rightarrow 2^{5x}=2^{-2}\Rightarrow 5x=-2\Rightarrow x=-\frac{2}{5}$

23. $\log_x 2=\frac{1}{5}\Rightarrow x^{\frac{1}{5}}=2\Rightarrow\left(x^{\frac{1}{5}}\right)^5=2^5\Rightarrow x^{\frac{5}{5}}=2^5\Rightarrow x=32$

B.4. Desarrollar expresiones logarítmicas:

24. $\log_a\frac{x\cdot y}{z}=\log_a x\cdot y-\log_a z=\log_a x+\log_a y-\log_a z$

25. $\log_a\left(\frac{x}{y}\right)^2=2\log_a\frac{x}{y}=2\left(\log_a x-\log_a y\right)$

26. $\log_a\frac{x\cdot y}{z}=\log_a x\cdot y-\log_a z=\log_a x+\log_a y-\log_a z$

27. $\log_a\frac{x^3y}{\sqrt{z}}=\log_a x^3y-\log_a\sqrt{z}=\log_a x^3+\log_a y-\log_a z^{\frac{1}{2}}=$

$$=3\log_a x+\log_a y-\frac{1}{2}\log_a z$$

B.5. Escribir como un solo logaritmo:

28. $\log(xy)-2\log\left(\frac{x}{y}\right)=\log(xy)-\log\left(\frac{x}{y}\right)^2=\log\left(\frac{xy}{\frac{x^2}{y^2}}\right)=\log\left(\frac{y^3}{x}\right)$

29. $2\ln(a-b)-\ln(a^2-b^2)=\ln(a-b)^2-\ln[(a+b)(a-b)]=$

$=\ln(a-b)^2-\ln[(a+b)(a-b)]=\ln\frac{(a-b)^{\not{2}}}{(a+b)\,\cancel{(a-b)}}=\ln\left(\frac{a-b}{a+b}\right)$

30. $4\log_2\frac{\sqrt{a-b}}{a}-\frac{1}{2}\log_2\left(\frac{a-b}{a}\right)^4=\log_2\left(\frac{\sqrt{a-b}}{a}\right)^4-\log_2\left[\left(\frac{a-b}{a}\right)^4\right]^{\frac{1}{2}}=$

$=\log_2\frac{(a-b)^2}{a^4}-\log_2\left(\frac{a-b}{a}\right)^2=\log_2\left[\frac{\frac{(a-b)^2}{a^4}}{\frac{(a-b)^2}{a^2}}\right]=\log_2\left[\frac{a^2(a-b)^2}{a^4(a-b)^2}\right]=$

$\log_2\left(\frac{1}{a^2}\right)=\log_2(a^{-2})$

31. $2\log_5(x)-\frac{1}{3}\log_5(b)+(x+2)\log_5(7)=\log_5 x^2-\log_5 b^{\frac{1}{3}}+\log_5 7^{x+2}=$

$=\log_5\frac{x^2}{b^{\frac{1}{3}}}+\log_5 7^{x+2}=\log_5\frac{x^2\cdot 7^{x+2}}{b^{\frac{1}{3}}}=\log_5\frac{x^2\cdot 7^{x+2}}{\sqrt[3]{b}}$

32. $\log\left(\frac{a}{b}\right)+\log\left(\frac{b}{c}\right)+\log\left(\frac{c}{d}\right)-\log\left(\frac{ay}{xd}\right)=\log\left(\frac{a}{\cancel{b}}\cdot\frac{\cancel{b}}{\cancel{c}}\cdot\frac{\cancel{c}}{d}\right)-\log\left(\frac{ay}{xd}\right)=$

$=\log\left(\frac{a}{cd}\right)-\log\left(\frac{ay}{xd}\right)=\log\left(\frac{\frac{a}{cd}}{\frac{ay}{xd}}\right)=\log\left(\frac{\frac{a}{cd}}{\frac{ay}{xd}}\right)=\log\left(\frac{x}{cy}\right)$

33. $\log_2(xy) - \log_2\left(\frac{x}{y^2}\right) + \frac{1}{2}\log_2\left(\frac{x^2y}{2}\right) =$

$$= \log_2\left(\frac{xy}{\frac{x}{y^2}}\right) + \log_2\left(\frac{x^2y}{2}\right) = \log_2\left(y^3\left(\frac{x^2y}{2}\right)\right) = \log_2\left(\frac{x^2y^4}{2}\right)$$

TEMAS RELACIONADOS

- Ecuaciones logarítmicas.
- Ecuaciones exponenciales.

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